ANALYSIS OF MICROWAVE SIGNAL RECEPTION USING FINITE DIFFERENCE IMPLEMENTATION. (A CASE STUDY OF AKURE – OWO DIGITAL MICROWAVE LINK IN SOUTH WESTERN NIGERIA)

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Abstract: In this work, the finite difference method have been used to model microwave signal propagation. The data used for this analysis were gathered between January and December 2006 in Akure – Owo digital microwave line of sight link in southwestern Nigeria. The data collected were analyzed using finite difference method and writing a program in MATLAB 7.0 software program to obtain a model equation for the line of sight link. The results of the work shows that the months of August, September, July and January have the poorest signal reception while the months of February, March and April have the best signal reception in the link. The results of the predicted model were validated by measured data and the results obtained showed that the developed model can be used to accurately predict the link degradation parameters.

Keywords: Microwave link, finite difference method, average signal level, signal reception.

INTRODUCTION

Justification for the work

Microwave signal transmission and reception in Nigeria especially for telephone services is very poor basically because of non-availability of data for planning and design of microwave links. There is a need for the build up of such a database in Nigeria [1]

Microwave signal propagation

Microwave radio relay is a technology for transmitting digital and analog signals such as long – distance telephone calls and the relay of television programs between two locations on a line of sight radio path [2, 3]

In a microwave radio relay, a line of sight link is required, therefore obstacles, the curvature of the earth, the geography of the area are important issues to consider when planning radio links.

Microwave propagation hardly occur under ideal conditions, for most communication links, the analysis must be modified to account for the presence of the earth, the ionosphere and atmosphere precipitates such as fog, raindrops, snow and hail, for stations on the ground transmitting through the lower atmosphere is complicated by uncontrolled variables associated with climate weather and path terrain. Signals are said to undergo fading which refers to the fact
that time – varying atmospheric processes influence the mechanisms of reflection, refraction and diffraction separately or in combination, so as to cause signal losses at a receiving antenna [3].

Once a microwave signal is radiated by the antenna, it will propagate through space and will ultimately reach the receiving antenna. As would be expected the energy level of the signal decreases rapidly as the distance from the transmitting antenna is increased further.

**Mathematical Modeling**

The finite difference method is a full wave method of parabolic equation that directly solve a wave equation numerically subject to a number of assumptions and simplifications. The finite difference method is based on discretisation of the wave equation through the introduction of a rectangular grid and the evaluation of the various derivative terms using centered finite differences [4,5,6,7,8].

The derivation of the parabolic equation normally start by reducing Maxwell’s equations to the Helmholtz equation. However in this instance if we assume the presence of an atmosphere described by a complex refractive index:

\[ n(r) = \sqrt{\varepsilon(r)} = \sqrt{\varepsilon_0 + j\sigma(r)\omega\varepsilon_0} \]  

which is a continuously varying function of position. Provided that \( \lambda_0 \ln n << 1 \), the scalar Helmholtz equation describes accurately each of the Cartesian components of the electric and magnetic fields.[,6,7]

\[ \nabla^2 \psi + k_0^2 n^2 \psi = 0 \]  

The wave number in vacuo is now given by \( k_0 = 2\pi / \lambda_0 \). Considering a two dimensional propagation problem along the great circle path and making the approximation that the earth is flat over a short length for simplicity, Equation 1.1 can be expanded in Cartesian co-ordinates as:

\[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k_0^2 n^2 \psi = 0 \]  

We now introduce the assumption that an a priori preferred direction of propagation exists and identify this with the \( x \) axis. It is, therefore, reasonable that we can write the following form for the solution: [6,7]

\[ \psi(x, z) = u(x, z) \exp(-jk_0x) \]  

where the reduced wave amplitude \( u(x, z) \) can now be assumed to vary slowly along the \( x \) direction on the scale of a free-space wavelength, \( \lambda \). Substituting Equation 1.3 into 1.2 and discarding the common factor \( \exp(-jk_0x) \) after performing the differentiations yields the following equation for the reduced wave amplitude:

\[ \frac{\partial^2 u}{\partial x^2} - 2jk_0 \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial z^2} + k_0^2 (n^2 - 1)u = 0 \]  

Equation 1.4 describes waves propagating both along the positive and negative \( x \) directions. By analogy with the one-dimensional wave equation

\[ \frac{\partial^2 w}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 w}{\partial t^2} = 0 \Rightarrow \left( \frac{\partial}{\partial x} - \frac{1}{c} \frac{\partial}{\partial t} \right) \left( \frac{\partial}{\partial x} + \frac{1}{c} \frac{\partial}{\partial t} \right) w = 0 \]  

which has linearly independent solutions given as:

\[ w = f(x - ct) \] and \( w = g(x + ct) \)  

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These can be identified as forward and backward traveling waves corresponding to the two differential operators in Equation 1.5a. If we factorise Equation 1.4 into forward and backward traveling wave operators; then we have

$$
\left( \frac{\partial}{\partial x} - j k_0 + j k_0 \sqrt{1 + (n^2 - 1)} + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right) \times
$$

$$
\left( \frac{\partial}{\partial x} - j k_0 - j k_0 \sqrt{1 + (n^2 - 1)} + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right) u = 0
$$

…………………(1.6)

If you discard the backward traveling wave for consistency with Equation 1.3, then finally, we have,

$$
\left( \frac{\partial}{\partial x} - j k_0 + j k_0 \sqrt{1 + (n^2 - 1)} + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right) u = 0 \quad \text{(1.7)}
$$

It is to be understood that the differential operator under the square root sign in Equation 1.7 can only be interpreted in a formal sense. Its numerical evaluation can only be achieved by replacing the square root by a power series, or rational fractions of operators.

Thus, we rewrite Equation 1.4 as:

$$
\frac{\partial u}{\partial x} = j k_0 \left( 1 - \sqrt{1 + (n^2 - 1)} + \frac{1}{k_0^2} \frac{\partial^2}{\partial z^2} \right) u = \sqrt{1 + Q(x, z)} u \quad \text{(1.8)}
$$

The differential operator $Q(x, z)$ must give a significantly smaller answer than the unity operator when operated on $u(x, z)$, since by assumption, the oscillatory variation of $i(x, z)$ is predominantly along the $x$ direction, perpendicular to the $z$ axis. Therefore, the $z$ derivative on a scale of a wavelength $(1/k_0)$ is much smaller than the unity operator. For the atmosphere, we also know that $n(x, z) \approx 1$, giving: [7]

$$
Q(x, z) u(x, z) \ll u(x, z) \quad \text{or formally} \quad Q \ll 1 \quad \text{………………………… (1.9)}
$$

The simplest approximation for the square root term is given by the first two terms in its Taylor expansion, namely,

$$
\sqrt{1 + Q(x, z)} \approx 1 + Q(x, z)/2 \quad \text{………………………… (1.10)}
$$

which finally yields the narrow-angle parabolic equation:

$$
\frac{\partial u}{\partial x} = \frac{1}{2 j k_0} \left( \frac{\partial^2 u}{\partial z^2} + k_0^2 (n^2 - 1) u \right) \quad \text{………………………… (1.11)}
$$

Parabolic Equation – Finite difference Implementation

This method is based on a more direct discretisation of Equation 1.11, through the introduction of a rectangular grid and the evaluation of the various derivative terms using centered finite differences. The various terms appearing in Equation 1.11 are evaluated at the centre point through their finite difference discrete approximations to yield; [7,8]
The line of sight microwave link used in this research work is situated between Akure located at latitude 071509.30N, longitude 0051142.60E (transmitting end) and Owo located at latitude 071220.00N longitude 0053402.00E (receiving end) over a path length of 41.42km. The Akure – Owo microwave link is owned and managed by NITEL – Nigeria Telecommunication Ltd. The microwave signal data were gathered between January and December 2006. The measurement was done with a data acquisition software PROCOMM PLUS 3.0 software program at the receiving end twice a week over a 24-hour period. This software was installed in a computer system (laptop) type 3050 Acer Aspire. The Laptop computer system was then connected to the NITEL equipment at Owo. The PROCOMM PLUS 3.0 software detects and captures the received signal values in the link. The system characteristics of the Akure – Owo digital microwave link is given in Table 1.0.

**Table 1.0** System characteristics of the Akure – Owo digital microwave link[9]

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Akure</th>
<th>Owo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation (M)</td>
<td>348</td>
<td>320</td>
</tr>
<tr>
<td>Latitude</td>
<td>071509.30N</td>
<td>071220.00N</td>
</tr>
<tr>
<td>Longitude</td>
<td>0051142.60E</td>
<td>0053402.00E</td>
</tr>
<tr>
<td>Antenna model</td>
<td>VHP4 – 71W</td>
<td>VHP 4 – 71W</td>
</tr>
<tr>
<td>Antenna Height (M)</td>
<td>90.00</td>
<td>90.00</td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>36.60</td>
<td>36.60</td>
</tr>
<tr>
<td>Frequency (MHz)</td>
<td>7500</td>
<td></td>
</tr>
<tr>
<td>Polarization</td>
<td>Vertical</td>
<td></td>
</tr>
<tr>
<td>Path length (km)</td>
<td>41.42</td>
<td></td>
</tr>
<tr>
<td>Radio Equipment model</td>
<td>MSM/H7 16E QPS</td>
<td></td>
</tr>
<tr>
<td>Transmitted power (dBm)</td>
<td>25.00 to 28.00</td>
<td></td>
</tr>
<tr>
<td>Main received signal (dBm)</td>
<td>45.03</td>
<td></td>
</tr>
<tr>
<td>Received Threshold level (dBm)</td>
<td>85.50</td>
<td></td>
</tr>
</tbody>
</table>

**RESULTS AND DISCUSSION**

The recorded microwave signal data for the period (January to December 2006) were computed into monthly averages as shown in Table 2.0.
Table 2.0: Akure – Owo Digital Microwave link, Year 2006 Average monthly data

<table>
<thead>
<tr>
<th>Month</th>
<th>Received Signal Level (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>- 47.0</td>
</tr>
<tr>
<td>February</td>
<td>- 36.0</td>
</tr>
<tr>
<td>March</td>
<td>- 37.0</td>
</tr>
<tr>
<td>April</td>
<td>- 36.0</td>
</tr>
<tr>
<td>May</td>
<td>- 39.0</td>
</tr>
<tr>
<td>June</td>
<td>- 39.0</td>
</tr>
<tr>
<td>July</td>
<td>- 48.0</td>
</tr>
<tr>
<td>August</td>
<td>- 50.0</td>
</tr>
<tr>
<td>September</td>
<td>- 50.0</td>
</tr>
<tr>
<td>October</td>
<td>- 46.0</td>
</tr>
<tr>
<td>November</td>
<td>- 40.0</td>
</tr>
<tr>
<td>December</td>
<td>- 47.0</td>
</tr>
</tbody>
</table>

(i) Variation of Average Signal level with months for Year 2006
The analysis of the results shows that the months of February, March and April have the best signal reception while the months of August, September and July have the poorest signal reception.

(ii) Analysis of Daily signal reception using the Finite difference model
The figure 1.0 shows the plot of the finite difference implementation of daily received signal level with distance.

![Figure 1.0: Daily microwave received signal level with distance](image)

Predicted Received Signal Level for Year 2006
The model equation is
\[ Y = 9 \times 10^{32} x^8 - 1.4 \times 10^{26} x^7 + 8.8 \times 10^{-22} x^6 \\
- 2.9 \times 10^{-17} x^5 + 5.4 \times 10^{13} x^4 - 5.5 \times 10^{19} x^3 \\
+ 2.8 \times 10^5 x^2 - 1.0 \times 10^5 x + 13 \quad \ldots \ldots \ldots (1.13) \]
If \( x \) (dBm) = Received signal level measured in equation (1.13) then \( Y \) (dBm) = predicted values.
For example, on Mondays 15th February 2006, the signal transmitted was 28dbm while the signal received was -35dBm. This corresponds to the model deduced as shown in equation 1.13 that if \(x = 28\text{dbm}\) then \(y = -35\text{dBm}\).

**CONCLUSION**

In this work microwave signal received level were measured on a monthly basis and the finite difference method was used to develop a model that can predict microwave signal received level on a daily basis.

The result of the research work shows that the months of August, September, July and January have poor signal reception while the months of February, march and April have good signal reception.

The model equation developed using the finite difference method is reasonably accurate.

**REFERENCES**


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